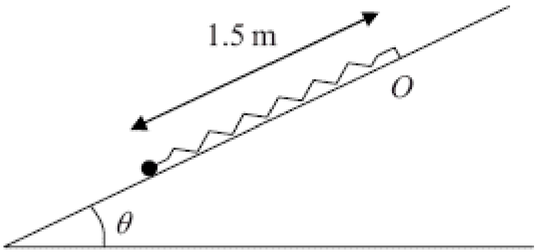
1.



A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in the diagram above. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ.

(Total 9 marks)

2. A light elastic string, of natural length $3a$ and modulus of elasticity $6mg$, has one end attached to a fixed point A. A particle P of mass $2m$ is attached to the other end of the string and hangs in equilibrium at the point O, vertically below A.

(a) Find the distance AO.

(3)

The particle is now raised to point C vertically below A, where $AC > 3a$, and is released from rest.

(b) Show that P moves with simple harmonic motion of period $2\pi\sqrt{\left(\frac{a}{g}\right)}$.

(5)

It is given that $OC = \frac{1}{4}a$.

(c) Find the greatest speed of P during the motion.

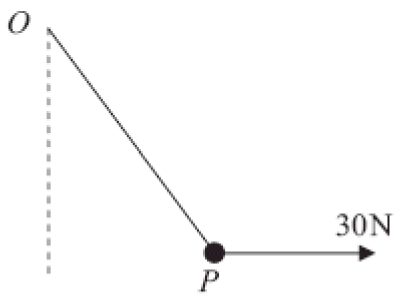
(3)

The point D is vertically above O and $OD = \frac{1}{8}a$. The string is cut as P passes through D, moving upwards.

(d) Find the greatest height of P above O in the subsequent motion.

(4)
(Total 15 marks)

3.



A particle P of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m. The other end of the string is attached to a fixed point O. A horizontal force of magnitude 30 N is applied to P, as shown in the diagram above. The particle P is in equilibrium and the elastic energy stored in the string is 10 J.

Calculate the length OP.

(Total 10 marks)

4. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A. The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v.

(a) Show that $v^2 = 2g(a + x) - \frac{3gx^2}{2a}$.

(4)

(b) Find the greatest speed attained by P as it falls.

(4)

After release, P next comes to instantaneous rest at a point D.

(c) Find the magnitude of the acceleration of P at D.

(6)
(Total 14 marks)

5. A light elastic string has natural length 8 m and modulus of elasticity 80 N.

The ends of the string are attached to fixed points P and Q which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below PQ.

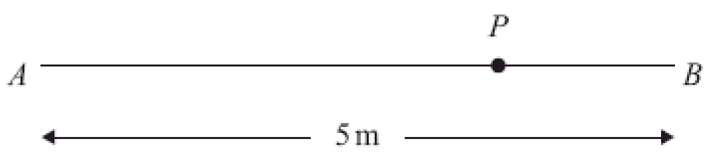
(a) Calculate the weight of the particle.

(6)

(b) Calculate the elastic energy in the string when the particle is in this position.

(3)
(Total 9 marks)

6.



A and B are two points on a smooth horizontal floor, where AB = 5 m.

A particle P has mass 0.5 kg. One end of a light elastic spring, of natural length 2 m and modulus of elasticity 16 N, is attached to P and the other end is attached to A. The ends of another light elastic spring, of natural length 1 m and modulus of elasticity 12 N, are attached to P and B, as shown in the diagram above.

(a) Find the extensions in the two springs when the particle is at rest in equilibrium.

(5)

Initially P is at rest in equilibrium. It is then set in motion and starts to move towards B. In the subsequent motion P does not reach A or B.

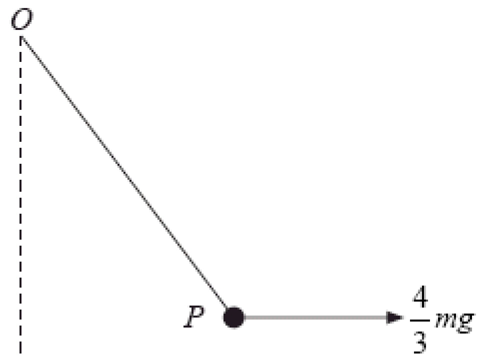
(b) Show that P oscillates with simple harmonic motion about the equilibrium position.

(4)

(c) Given that the initial speed of P is $\sqrt{10}$ m s^{-1}, find the proportion of time in each complete oscillation for which P stays within 0.25 m of the equilibrium position.

(7)
(Total 16 marks)

7.



A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O.

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P.

This force acts in the vertical plane containing the string, as shown in the diagram above. Find

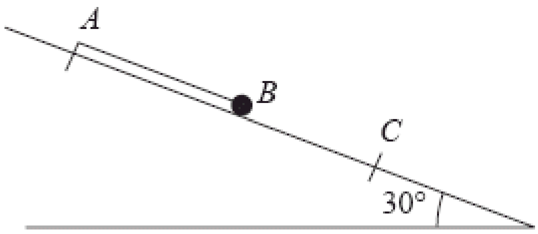
(a) the tension in the string,

(5)

(b) the elastic energy stored in the string.

(4)
(Total 9 marks)

8.



One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in the diagram above.
Find

(a) the length AC,

(5)

(b) the greatest speed attained by B as it moves from its initial position to C.

(7)
(Total 12 marks)

9. A light elastic string of natural length 0.4 m has one end A attached to a fixed point. The other end of the string is attached to a particle P of mass 2 kg. When P hangs in equilibrium vertically below A, the length of the string is 0.56 m.

(a) Find the modulus of elasticity of the string.

(3)

A horizontal force is applied to P so that it is held in equilibrium with the string making an angle θ with the downward vertical. The length of the string is now 0.72 m.

(b) Find the angle θ.

(3)
(Total 6 marks)

10. A particle P of mass m lies on a smooth plane inclined at an angle 30° to the horizontal. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point O on the plane. The particle P is in equilibrium at the point A on the plane and the extension of the string is $\frac{1}{4}a$.

The particle P is now projected from A down a line of greatest slope of the plane with speed V. It comes to instantaneous rest after moving a distance $\frac{1}{2}a$.

By using the principle of conservation of energy,

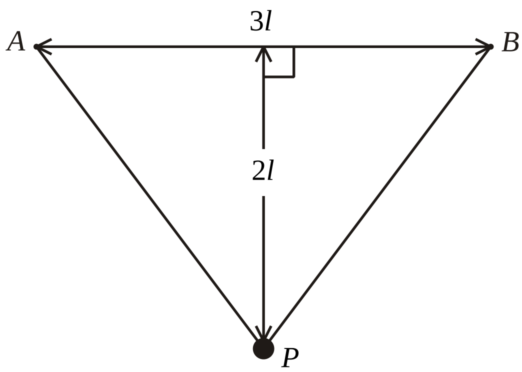
(a) find V in terms of a and g,

(6)

(b) find, in terms of a and g , the speed of P when the string first becomes slack.

(4)
(Total 10 marks)

11.



A light elastic string, of natural length $3l$ and modulus of elasticity λ, has its ends attached to two points A and B, where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB, as shown in the diagram above,

(a) show that $\lambda = \frac{15mg}{16}$.

(9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

(b) Show that P comes to instantaneous rest on the line AB.

(6)
(Total 15 marks)

12. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3.6mg$. The other end of the string is fixed at a point O on a rough horizontal table. The particle is projected along the surface of the table from O with speed $\sqrt{(2ag)}$. At its furthest point from O, the particle is at the point A, where $OA = \frac{4}{3}a$.

(a) Find, in terms of m, g and a, the elastic energy stored in the string when P is at A.

(3)

(b) Using the work-energy principle, or otherwise, find the coefficient of friction between P and the table.

(6)
(Total 9 marks)

13. A particle P of mass 0.25 kg is attached to one end of a light elastic string. The string has natural length 0.8 m and modulus of elasticity λ N. The other end of the string is attached to a fixed point A. In its equilibrium position, P is 0.85 m vertically below A.

(a) Show that $\lambda = 39.2$.

(2)

The particle is now displaced to a point B, 0.95 m vertically below A, and released from rest.

(b) Prove that, while the string remains stretched, P moves with simple harmonic motion of period $\frac{\pi}{7}$ s.

(6)

(c) Calculate the speed of P at the instant when the string first becomes slack.

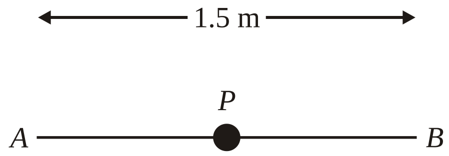
(3)

The particle first comes to instantaneous rest at the point C.

(d) Find, to 3 significant figures, the time taken for P to move from B to C.

(5)
(Total 16 marks)

14. Two light elastic strings each have natural length 0.75 m and modulus of elasticity 49 N. A particle P of mass 2 kg is attached to one end of each string. The other ends of the strings are attached to fixed points A and B, where AB is horizontal and AB = 1.5 m.



The particle is held at the mid-point of AB. The particle is released from rest, as shown in the figure above.

(a) Find the speed of P when it has fallen a distance of 1 m.

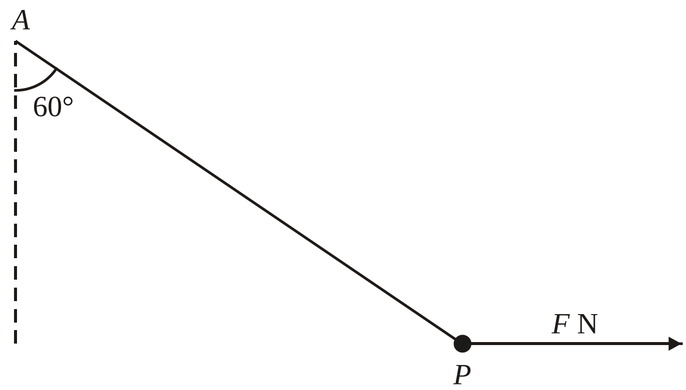
(6)

Given instead that P hangs in equilibrium vertically below the mid-point of AB, with $\angle APB = 2\alpha$,

(b) show that $\tan\alpha + 5\sin\alpha = 5$.

(6)
(Total 12 marks)

15.



A particle P of mass 0.8 kg is attached to one end of a light elastic string, of natural length 1.2 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point A. A horizontal force of magnitude F newtons is applied to P. The particle P is in equilibrium with the string making an angle 60° with the downward vertical, as shown in the figure above.

Calculate

(a) the value of F,

(3)

(b) the extension of the string,

(3)

(c) the elastic energy stored in the string.

(2)
(Total 8 marks)

16. A light elastic string of natural length l has one end attached to a fixed point A. A particle P of mass m is attached to the other end of the string and hangs in equilibrium at the point O, where $AO = \frac{5}{4}l$.

(a) Find the modulus of elasticity of the string.

(2)

The particle P is then pulled down and released from rest. At time t the length of the string is $\frac{5l}{4} + x$.

(b) Prove that, while the string is taut,

$$\frac{d^2x}{dt^2} = -\frac{4gx}{l}$$

(5)

When P is released, $AP = \frac{7}{4}l$. The point B is a distance l vertically below A.

(c) Find the speed of P at B.

(4)

(d) Describe briefly the motion of P after it has passed through B for the first time until it next passes through O.

(2)
(Total 13 marks)

17. A light elastic string has natural length $2l$ and modulus of elasticity $4mg$. One end of the string is attached to a fixed point A and the other end to a fixed point B, where A and B lie on a smooth horizontal table, with $AB = 4l$. A particle P of mass m is attached to the mid-point of the string. The particle is released from rest at the point of the line AB which is $\frac{5l}{3}$ from B. The speed of P at the mid-point of AB is V.

(a) Find V in terms of g and l.

(7)

(b) Explain why V is the maximum speed of P.

(2)
(Total 9 marks)

18. A light spring of natural length L has one end attached to a fixed point A. A particle P of mass m is attached to the other end of the spring. The particle is moving vertically. As it passes through the point B below A, where $AB = L$, its speed is $\sqrt{(2gL)}$. The particle comes to instantaneous rest at a point C, $4L$ below A.

(a) Show that the modulus of elasticity of the spring is $\dfrac{8mg}{9}$.

(4)

At the point D the tension in the spring is mg.

(b) Show that P performs simple harmonic motion with centre D.

(5)

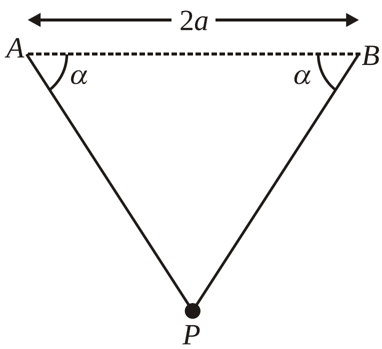
(c) Find, in terms of L and g,

(i) the period of the simple harmonic motion,

(ii) the maximum speed of P.

(5)
(Total 14 marks)

19.

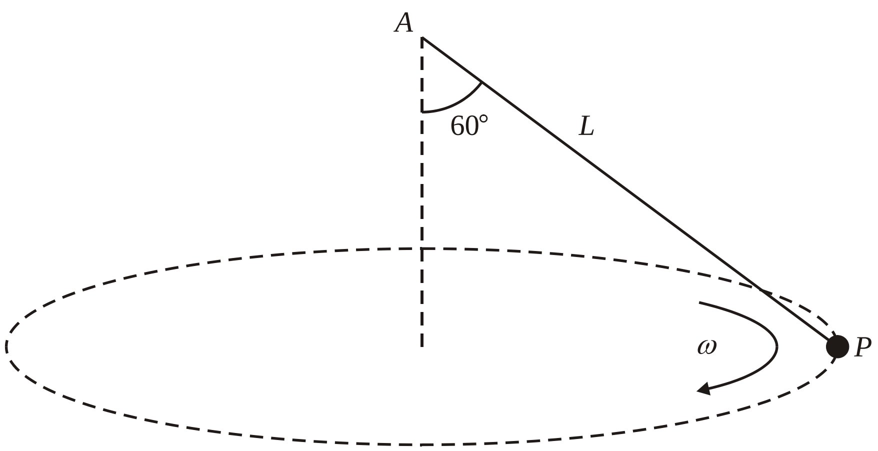


Two light elastic strings each have natural length a and modulus of elasticity λ. A particle P of mass m is attached to one end of each string. The other ends of the strings are attached to points A and B, where AB is horizontal and $AB = 2a$. The particle is held at the mid-point of AB and released from rest. It comes to rest for the first time in the subsequent motion when PA and PB make angles α with AB, where $\tan \alpha = \frac{4}{3}$, as shown in the diagram above.

Find λ in terms of m and g.

(Total 7 marks)

20.



A particle P of mass m is attached to one end of a light string. The other end of the string is attached to a fixed point A. The particle moves in a horizontal circle with constant angular speed ω and with the string inclined at an angle of 60° to the vertical, as shown in the diagram above. The length of the string is L.

(a) Show that the tension in the string is $2mg$.

(1)

(b) Find ω in terms of g and L.

(4)

The string is elastic and has natural length $\frac{3}{5}L$.

(c) Find the modulus of elasticity of the string.

(2)
(Total 7 marks)

21. A particle P of mass m is attached to one end of a light elastic string of length a and modulus of elasticity $\frac{1}{2}mg$. The other end of the string is fixed at the point A which is at a height $2a$ above a smooth horizontal table. The particle is held on the table with the string making an angle β with the horizontal, where $\tan \beta = \frac{3}{4}$.

(a) Find the elastic energy stored in the string in this position.

(3)

The particle is now released. Assuming that P remains on the table,

(b) find the speed of P when the string is vertical.

(3)

By finding the vertical component of the tension in the string when P is on the table and AP makes an angle θ with the horizontal,

(c) show that the assumption that P remains in contact with the table is justified.

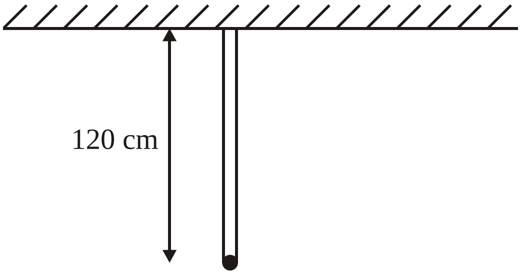
(5)
(Total 11 marks)

22. A particle P of mass m is held at a point A on a rough horizontal plane. The coefficient of friction between P and the plane is $\frac{2}{3}$. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $4mg$. The other end of the string is attached to a fixed point O on the plane, where $OA = \frac{3}{2}a$. The particle P is released from rest and comes to rest at a point B, where $OB < a$.

Using the work-energy principle, or otherwise, calculate the distance AB.

(Total 6 marks)

23.



A particle of mass 5 kg is attached to one end of two light elastic strings. The other ends of the strings are attached to a hook on a beam. The particle hangs in equilibrium at a distance 120 cm below the hook with both strings vertical, as shown in the diagram above. One string has natural length 100 cm and modulus of elasticity 175 N. The other string has natural length 90 cm and modulus of elasticity λ newtons.

Find the value of λ.

(Total 5 marks)

24. A light elastic string has natural length 4 m and modulus of elasticity 58.8 N. A particle *P* of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a fixed point *A*. The particle is released from rest at *A* and falls vertically.

(a) Find the distance travelled by *P* before it comes to instantaneous rest for the first time.

(7)

The particle is now held at a point 7 m vertically below *A* and released from rest.

(b) Find the speed of the particle when the string first becomes slack.

(5)
(Total 12 marks)

25. A light elastic string AB has one end A attached to a fixed point on a ceiling. A particle P of mass 0.3 kg is attached to B. When P hangs in equilibrium with AB vertical, AB = 100 cm. The particle P is replaced by another particle Q of mass 0.5 kg. When Q hangs in equilibrium with AB vertical, AB = 110 cm. Find

(a) the natural length of the string,

(5)

(b) the modulus of elasticity of the string.

(2)
(Total 7 marks)

26. In a "test your strength" game at an amusement park, competitors hit one end of a small lever with a hammer, causing the other end of the lever to strike a ball which then moves in a vertical tube whose total height is adjustable. The ball is attached to one end of an elastic spring of natural length 3 m and modulus of elasticity 120 N. The mass of the ball is 2 kg. The other end of the spring is attached to the top of the tube. The ball is modelled as a particle, the spring as light and the tube is assumed to be smooth.

The height of the tube is first set at 3 m. A competitor gives the ball an initial speed of 10 m s^{-1}.

(a) Find the height to which the ball rises before coming to rest.

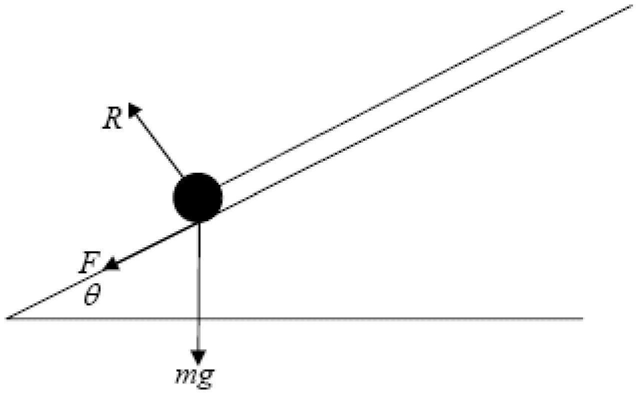
(6)

The tube is now adjusted by reducing its height to 2.5 m. The spring and the ball remain unchanged.

(b) Find the initial speed which the ball must now have if it is to rise by the same distance as in part (*a*).

(5)
(Total 11 marks)

1.



EPE lost $= \dfrac{\lambda \times 0.6^2}{2 \times 0.9} - \dfrac{\lambda \times 0.1^2}{2 \times 0.9} \left(= \dfrac{7}{36}\lambda \right)$ M1 A1

R$(\uparrow)$ $R = mg\cos\theta$ M1

$= 0.5g \times \dfrac{4}{5} = 0.4g$

$F = \mu R = 0.15 \times 0.4g$ M1 A1

P.E. gained = E.P.E. lost − work done against friction

$0.5\text{g} \times 0.7\sin\theta = \dfrac{\lambda \times 0.6^2}{2 \times 0.9} - \dfrac{\lambda \times 0.1^2}{2 \times 0.9} - 0.15 \times 0.4g \times 0.7$ M1 A1 A1

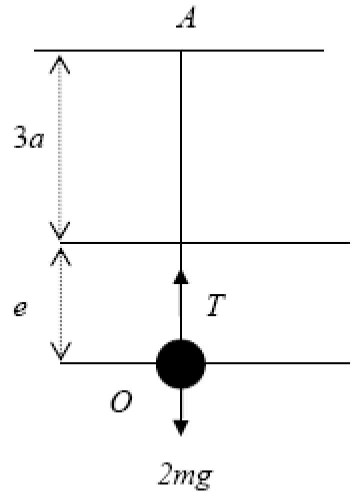
$0.1944\lambda = 0.5 \times 9.8 \times 0.7 \times \dfrac{3}{5} + 0.15 \times 0.4 \times 9.8 \times 0.7$

$\lambda = 12.70......$

$\lambda = 13$ N or 12.7 A1

[9]

2. (a)



$\text{R}(\uparrow) \quad T = 2mg$ B1

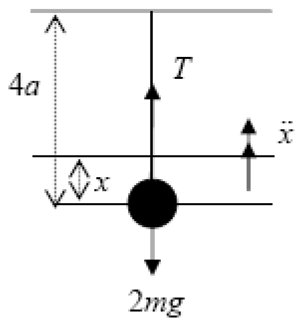
Hooke's law: $T = \dfrac{6mge}{3a}$

$$2mg = \frac{6mge}{3a}$$ M1

$$e = a$$

$$AO = 4a$$ A1 3

(b)



H.L. $T = \dfrac{6mg(a-x)}{3a} = \dfrac{2mg(a-x)}{a}$ B1ft

Eqn. of motion $-2mg + T = 2m\ddot{x}$ M1

$$-2mg + \frac{2mg(a-x)}{a} = 2m\ddot{x}$$ M1

$$-\frac{2mgx}{a} = 2m\ddot{x}$$

$$\ddot{x} = -\frac{g}{a}x$$ A1

period $2\pi\sqrt{\dfrac{a}{g}}$ * A1 5

(c) $$v^2 = \omega^2\left(a^2 - x^2\right)$$

$$v_{\max}{}^2 = \frac{g}{a}\left(\left(\frac{a}{4}\right)^2 - 0\right)$$ M1 A1

$$v_{\max} = \frac{1}{4}\sqrt{(ga)}$$ A1 3

(d) $x = -\frac{a}{8}$ $$v^2 = \frac{g}{a}\left(\frac{a^2}{16} - \frac{a^2}{64}\right)$$ M1

$$= \frac{3ag}{64}$$

$$v^2 = u^2 + 2as$$ M1

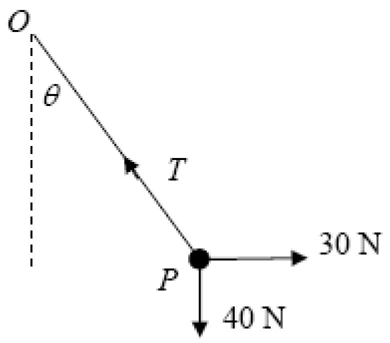
$$0 = \frac{3ag}{64} - 2gh$$ A1

$$h = \frac{3a}{128}$$

Total height above $O = \frac{a}{8} + \frac{3a}{128} = \frac{19a}{128}$ A1 4

[15]

3.



↑	$T\cos\theta = 40$	M1 attempt at both equations	M1 A1
→	$T\sin\theta = 30$		A1
leading to	$T = 50$		M1 A1
	$E = \dfrac{\lambda x^2}{2a} = 10$		B1
HL	$T = \dfrac{\lambda x}{a} = 50$		M1
leading to	$x = 0.4$		M1 A1
	$OP = 0.5+0.4=0.9$ (m)		A1ft

[10]

4. (a) $\dfrac{1}{2}mv^2 + \dfrac{3mgx^2}{4a} = mg(a+x)$ M1 A2 (1, 0)

leading to $v^2 = 2g(a+x) - \dfrac{3gx^2}{2a}$ * cso A1 4

(b) Greatest speed is when the acceleration is zero

$T = \dfrac{\lambda x}{a} = \dfrac{3mgx}{2a} = mg \Rightarrow x = \dfrac{2a}{3}$ M1 A1

$v^2 = 2g\left(a + \dfrac{2a}{3}\right) - \dfrac{3g}{2a} \times \left(\dfrac{2a}{3}\right)^2 \left(= \dfrac{8ag}{3}\right)$ M1

$v = \dfrac{2}{3}\sqrt{(6ag)}$ accept exact equivalents A1 4

Alternative

$$v^2 = 2g(a+x) - \frac{3gx^2}{2a}$$

Differentiating with respect to x

$$2v\frac{\mathrm{d}v}{\mathrm{d}x} = 2g - \frac{3gx}{a}$$

$$\frac{\mathrm{d}v}{\mathrm{d}x} = 0 \Rightarrow x = \frac{2a}{3}$$ M1 A1

$$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$$ M1

$$v = \frac{2}{3}\sqrt{(6ag)}$$ accept exact equivalents A1

(c) $v = 0 \Rightarrow 2g(a+x) - \dfrac{3gx^2}{2a} = 0$ M1

$3x^2 - 4ax - 4a^2 = (x - 2a)(3x + 2a) = 0$

$x = 2a$ M1 A1

At D, $m\ddot{x} = mg - \dfrac{\lambda \times 2a}{a}$ ft their $2a$ M1 A1ft

$|\ddot{x}| = 2g$ A1 6

Alternative approach using SHM for (b) and (c)

If SHM is used mark (b) and (c) together placing the marks in the gird as shown.

Establishment of equilibrium position

$$T = \frac{\lambda x}{a} = \frac{3mge}{2a} = mg \Rightarrow e = \frac{2a}{3}$$ bM1 bA1

N2L, using y for displacement from equilibrium position

$$m\ddot{y} = mg - \frac{\frac{3}{2}mg(y+e)}{a} = -\frac{3g}{2a}y$$ bM1 bA1

$$\omega^2 = \frac{3g}{2a}$$

Speed at end of free fall $u^2 = 2ga$ cM1

Using A for amplitude and $v^2 = \omega^2 (a^2 - x^2)$

$$u^2 = 2ga \text{ when } y = -\frac{2}{3}a \Rightarrow 2ga = \frac{3g}{2a}\left(A^2 - \frac{4a^2}{9}\right)$$ cM1

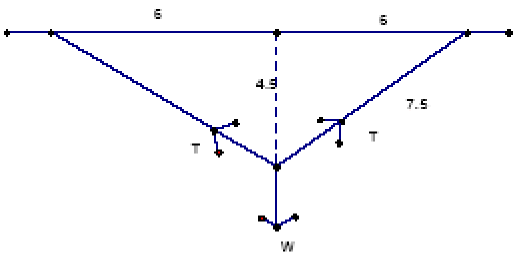
$$A = \frac{4a}{3}$$ cA1

Maximum speed $A\omega = \frac{4a}{3} \times \sqrt{\left(\frac{3g}{2a}\right)} = \frac{2}{3}\sqrt{(6ag)}$ cM1 cA1

Maximum acceleration $A\omega^2 = \frac{4a}{3} \times \frac{3g}{2a} = 2g$ cA1

[14]

5. (a)



Resolving vertically: $2T\cos\theta = W$ M1A2,1,0

Hooke's Law: $T = \frac{80 \times 3.5}{4}$ M1A1

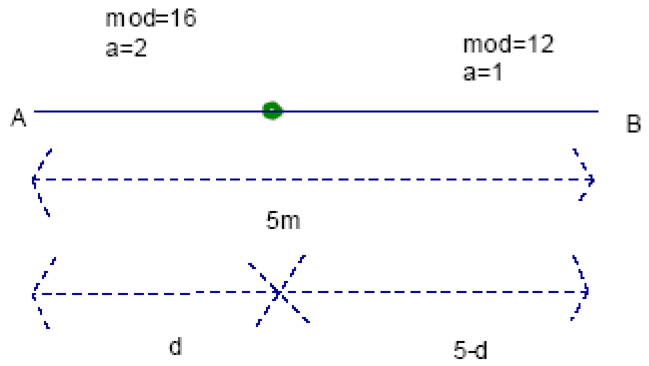
$W = 84\text{N}$ A1

(b) EPE = $2 \times \frac{80 \times 3.5^2}{2 \times 4}$, =245 (or awrt 245) M1A1ft,A1

(alternative $\frac{80 \times 7^2}{16} = 245$)

[9]

6. (a)



Hooke's law: Equilibrium $\Rightarrow \dfrac{16(d-2)}{2} = \dfrac{12(4-d)}{1}$ M1A1A1

$\Rightarrow\ d = 3.2$ A1

so extensions are 1.2m and 0.8m. A1

(b) If the particle is displaced distance x towards ***B*** then M1A1ft

$-m\ddot{x} = \dfrac{16(1.2+x)}{2} - \dfrac{12(0.8-x)}{1} (= 20x)$ A1ft

$\Rightarrow \ddot{x} = -40x \text{ or } \ddot{x} = -\dfrac{20}{m} (\Rightarrow \text{SHM})$ A1

(c) $T = \dfrac{2\pi}{\sqrt{40}}$ B1ft

$a = \dfrac{\sqrt{10}}{\text{their}\,\omega}$ B1ft

$x = a \sin \omega t$ their a, their ω M1

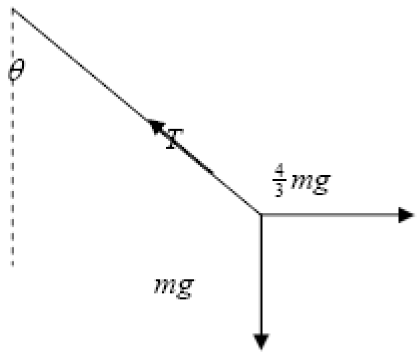
$\dfrac{1}{4} = \dfrac{1}{2} \sin \sqrt{40}t$ A1

$\sqrt{40}t = \dfrac{\pi}{6} (\Rightarrow t = \dfrac{\pi}{6\sqrt{40}})$ M1

Proportion $\dfrac{4t}{T} = \dfrac{4\pi}{6\sqrt{40}} \times \dfrac{\sqrt{40}}{2\pi} = \dfrac{1}{3}$ M1A1

[16]

7. (a)



$(\leftarrow)$	$T\sin\theta = \frac{4}{3mg}$	M1 A1	
$(\uparrow)$	$T\cos\theta = mg$	A1	
	$T^2 = \left(\frac{4}{3}mg\right)^2 + (mg)^2$	M1	
Leading to	$T = \frac{5}{3}mg$	A1	5

(b) HL $T = \frac{\lambda x}{a} \Rightarrow \frac{5}{3}mg = \frac{3mge}{a}$ ft their T M1 A1ft

$e = \frac{5}{9}a$

$E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9}a\right)^2 = \frac{25}{54}mga$ M1 A1 4

[9]

8. (a) Let x be the distance from the initial position of B to C

GPE lost = EPE gained

$mgx\sin 30^\circ = \frac{6mgx^2}{2a}$ M1 A1 = A1

Leading to $x = \frac{a}{6}$ M1

$AC = \frac{7a}{6}$ A1 5

(b) The greatest speed is attained when the acceleration of B is zero, that is where the forces on B are equal.

(↖) $T = mg\sin 30^\circ = \dfrac{6\text{mge}}{\text{a}}$ M1

$e = \dfrac{a}{12}$ A1

CE $\dfrac{1}{2}mv^2 + \dfrac{6mg}{2a}\left(\dfrac{a}{12}\right)^2 = mg\dfrac{a}{12}\sin 30^\circ$ M1 A1 = A1

Leading to $v = \sqrt{\left(\dfrac{ga}{24}\right)} = \dfrac{\sqrt{6ga}}{12}$ M1 A1 7

Alternative approach to (b) *using calculus with energy.*

Let distance moved by B be x

CE $\dfrac{1}{2}mv^2 + \dfrac{6mg}{2a}x^2 = mgx\sin 30^\circ$ M1 A1 = A1

$v^2 = gx - \dfrac{6g}{a}x^2$

For maximum v $\dfrac{\mathrm{d}}{\mathrm{d}x}(v^2) = 2v\dfrac{\mathrm{d}v}{\mathrm{d}x} = g - \dfrac{12g}{a}x = 0$ M1 A1

$x = \dfrac{a}{12}$

$v^2 = g\left(\dfrac{a}{12}\right) - \dfrac{6g}{a}\left(\dfrac{a}{12}\right)^2 = \dfrac{ga}{24}$ M1

$v = \sqrt{\left(\dfrac{ga}{24}\right)}$ A1 7

Alternative approach to (b) *using calculus with Newton's second law.*

As before, the centre of the oscillation is when extension is $\frac{a}{12}$ M1 A1

N2L $mg \sin 30° - T = m\ddot{x}$

$$\frac{1}{2}mg - \frac{6mg\left(\frac{a}{12}+x\right)}{a} = m\ddot{x}$$ M1 A1

$$\ddot{x} = -\frac{6g}{a}x \Rightarrow \omega^2 = \frac{6g}{a}$$ A1

$$v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a}\right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24}\right)}$$ M1 A1 7

[12]

9. (a) T or $\frac{\lambda \times e}{l}$ = mg (even $T = m$ is M1, A0, A0 sp case) M1

$\frac{\lambda \times 0.16}{0.4}$ = $2g$ A1

$\Rightarrow \lambda$ = <u>49 N</u> or 5g A1 3

(b) R(↑) $T \cos \theta = mg$ or $\cos \theta = \frac{mg}{T}$ M1

$49.\frac{0.32}{0.4}.\cos\theta = 19.6$ or $4g.\cos\theta = 2g$ or $2mg.\cos\theta = mg$ (ft on their λ)A1ft

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$ (or $\frac{\pi}{3}$ radians) A1 3

Special case $T \sin \theta = mg$ giving $\theta = 30$ is M1 A0 A0 unless
there is evidence that they think θ is with horizontal – then M1 A1 A0

[6]

10. (a) Energy equation with at least three terms, including K.E term M1

$\frac{1}{2}mV^2 + ...$

$+..\frac{1}{2}.\frac{2mg}{a}.\frac{a^2}{16}, +mg.\frac{1}{2}a.\sin 30, = \frac{1}{2}.\frac{2mg}{a}.\frac{9a^2}{16}$ A1, A1, A1

$\Rightarrow V = \sqrt{\frac{ga}{2}}$ dM1A1 6

(b) Using point where velocity is zero and point where string becomes slack: M1

$\frac{1}{2}mw^2 =$

$\frac{1}{2}.\frac{2mg}{a}.\frac{9a^2}{16}, -mg.\frac{3a}{4}.\sin 30$ A1, A1

$\Rightarrow w = \sqrt{\frac{3ag}{8}}$ A1 4

Alternative (using point of projection and point where string becomes slack): M1, A1A1

$\frac{1}{2}mw^2 - \frac{1}{2}mV_1^2, = \frac{mga}{16} - \frac{mga}{8}$ A1

So $w = \sqrt{\frac{3ag}{8}}$

In part (a)
DM1 requires EE, PE and KE to have been included in the energy equation.

If sign errors lead to $V^2 = -\frac{ga}{2}$, the last two marks are M0 A0

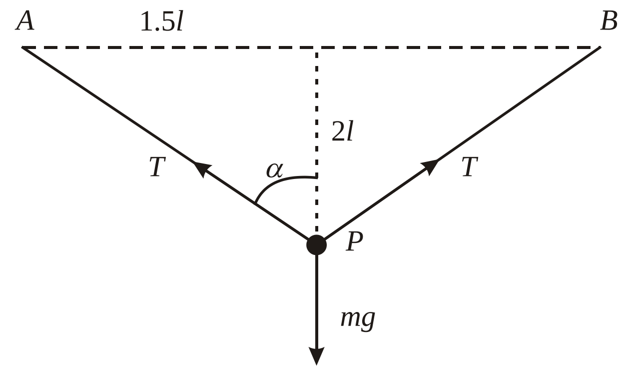
In parts (a) and (b) A marks need to have the correct signs
In part (b) for M1 need **one** KE term in energy equation of at least **3 terms** with distance $\frac{3a}{4}$ to indicate first method, and **two** KE terms in energy equation of at least **4 terms** with distance $\frac{a}{4}$ to indicate second method.

SHM approach in part (b). (**Condone this method only if SHM is proved**)
Using $v^2 = \omega^2(a^2 - x^2)$ with $\omega^2 = \frac{2g}{a}$ and $x = \pm\frac{a}{4}$.

Using 'a' = $\frac{a}{2}$ to give $w = \sqrt{\frac{3ag}{8}}$.

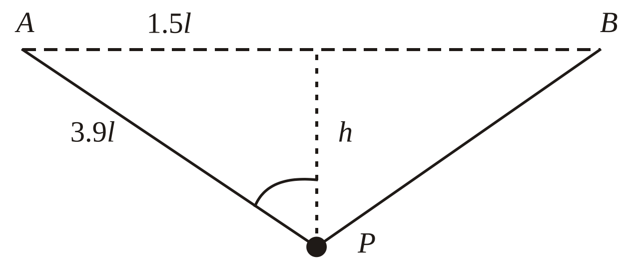
[10]

11. (a)



$AP = \sqrt{((1.5l)^2 + (2l)^2)} = 2.5l$		M1A1	
$\cos\alpha = \frac{4}{5}$		B1	
Hooke's Law $T = \frac{\lambda(2.5l - 1.5l)}{1.5l} \left(= \frac{2\lambda}{3}\right)$		M1A1	
$\uparrow 2T\cos\alpha = mg \quad \left(T = \frac{5mg}{8}\right)$		M1A1	
$2 \times \frac{2\lambda}{3} \times \frac{4}{5} = mg \left(\frac{2\lambda}{3} = \frac{5mg}{8}\right)$		M1	
$\lambda = \frac{15mg}{16}$ *	cso	A1	9

(b)



$h = \sqrt{\left((3.9l)^2 - (1.5l)^2\right)} = 3.6l$		M1A1	
Energy $\frac{1}{2}mv^2 + mg \times h = 2 \times \frac{15mg}{16} \times \frac{(2.4l)^2}{2 \times 1.5l}$	ft their h	M1A1ft=A1	
Leading to $v = 0$ *	cso	A1	6

[15]

12. (a) E.P.E. $= \frac{1}{2}\frac{3.6mg}{a}x^2 = \frac{1}{2}\frac{3.6mg}{a}\left(\frac{a}{3}\right)^2$		M1A1	
$= \underline{0.2\ mga}$		A1	3

(b) Friction = $\mu mg \Rightarrow$ work done by friction = $\mu mg\left(\dfrac{4a}{3}\right)$ M1A1

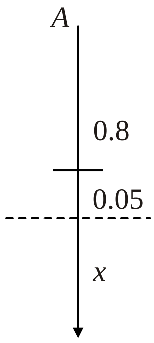
Work-energy: $\dfrac{1}{2}m.2ga = \mu mgd + 0.2\,mga$ (3 relevant terms) M1A1ft

Solving to find μ : $\underline{\mu = 0.6}$ M1A1 6

1st M1: allow for attempt to find work done by frictional force
(i.e. not just finding friction).
2nd M1: "relevant" terms, i.e. energy or work terms!
A1 f.t. on their work done by friction

[9]

13. (a)



$T = \dfrac{\lambda}{0.8}(0.05) = 0.25\text{g}$ M1

$\lambda = \dfrac{(0.8)(0.25\text{g})}{0.05} = 39.2$ (*) A1 2

(b) $T = \dfrac{39.2}{0.8}(x + 0.05)$ M1

$mg - T = ma$ (3 term equn) M1

$0.25\text{g} - \dfrac{39.2}{0.8}(x + 0.05) = 0.25\ \ddot{x}$ (or equivalent) A1

$\ddot{x} = -196\,x$ A1

SHM with period $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{14} = \dfrac{\pi}{7}$ s (*) M1 A1 cso 6

1st M1 must have extn as $x + k$ with $k \neq 0$ (but allow M1 if
e.g. $x + 0.15$), or must justify later

For last four marks, *must* be using $\ddot{x}$ (not a)

(c) $v = 14\sqrt{\{(0.1)^2 - (0.05)^2\}}$ M1 A1ft

$= 1.21(24\ldots) \approx \underline{1.21 \text{ m s}^{-1}}$ (3 s.f.) Accept $7\sqrt{3}/10$ A1 3

Using $x = 0$ is M0

(d) Time T under gravity = $\frac{1.21..}{g}$ (= 0.1237 s) B1ft

Complete method for time T' from B to slack.

[↑ e.g. $\frac{\pi}{28}$ + t, where 0.05 = 0.1sin 14t M1 A1

OR T', where – 0.05 = 0.1 cos 14 T']

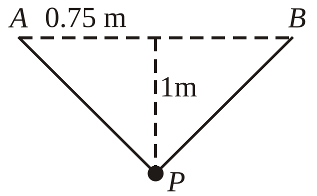
T'' = 0.1496s A1

Total time = $T + T'$ = <u>0.273 s</u> A1 5

M1 – must be using distance for when string goes slack.
Using x = –0.1 (i.e. assumed end of the oscillation) is M0

[16]

14. (a)



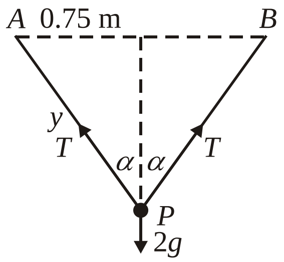
$AP = \sqrt{(0.75^2 + 1^2)} = 1.25$ M1 A1

Conservation of energy

$\frac{1}{2}\times 2\times v^2 + 2\times\frac{49\times 0.5^2}{2\times 0.75} = 2g\times 1$ −1 for each incorrect term M1 A2 (1, 0)

Leading to $v \approx 1.8$ (ms^{-1}) accept 1.81 A1 6

(b)



R (↑)	$2T\cos\alpha = 2g$		M1 A1	
	$y = \dfrac{0.75}{\sin\alpha}$			
Hooke's Law	$T = \dfrac{49}{0.75}\left(\dfrac{0.75}{\sin\alpha} - 0.75\right)$		M1 A1	
	$= 49\left(\dfrac{1}{\sin\alpha} - 1\right)$			
	$\dfrac{9.8}{\cos\alpha} = 49\left(\dfrac{1}{\sin\alpha} - 1\right)$	Eliminating T	M1	
	$\tan\alpha = 5\,(1 - \sin\alpha)$			
	$5 = \tan\alpha + 5\sin\alpha$	cso	A1	6

[12]

15. (a) $F = T\sin 60°$ i $T\cos 60° = 0.8g$ both M1

[or Z $F\cos 60° = 0.8g\cos 30°$] (M2)

$F = 0.8g\tan 60° \approx 14$(N) accept 13.6 M1 A1 3

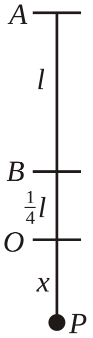
(b) $T = \dfrac{0.8g}{\sin 30°}$ (= 15.68) allow in (a) M1

HL $15.68 = \dfrac{24 \times x}{1.2}$ ⇒ $x \approx 0.78$ (cm) accept 0.784 M1 A1 3

c) $E = \dfrac{24 \times x^2}{2 \times 1.2} \approx 6.1$ (J) accept 6.15 M1 A1ft 2

[8]

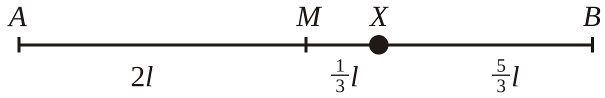
16.



(a)	HL $\quad T = mg = \dfrac{\lambda \times \frac{1}{4}l}{l} \Rightarrow \quad \lambda = 4mg$		M1 A1	2
(b)	N2L $\quad mg - T - \; m\ddot{x}$		M1	
	$mg - \dfrac{4mg(\frac{1}{4}l + x)}{l} = m\ddot{x}$		M1 A1	
	$\dfrac{d^2x}{dt^2} = -\dfrac{4g}{l}x$	cso	M1 A1	5
(c)	$v^2 - \omega^2(a^2 - x^2) = \dfrac{4g}{l}\left(\dfrac{l^2}{4} - \dfrac{l^2}{16}\right)$		M1 A1	
	Leading to $v = \dfrac{1}{2}\sqrt{(3gl)}$		M1 A1	4
	or energy, $\dfrac{1}{2}\dfrac{4mg.gl^2/16}{l} = \dfrac{1}{2}mv^2 + mg.\dfrac{3l}{4}$ for the first M1 A1 in (c)			
(d)	P first moves freely under gravity,		B1	
	then (part) SHM.		B1	2

[13]

17. (a)



Elastic energy when P is at X: $E = \dfrac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \dfrac{4mg\left(\frac{4}{3}l\right)^2}{2l}\left(= \dfrac{40mgl}{9}\right)$ M1 A1

$\dfrac{1}{2}mV^2 + 2\times\dfrac{4mgl^2}{2l} = \dfrac{4mh\left(\frac{2}{3}l\right)^2}{2l} + \dfrac{4mg\left(\frac{4}{3}l\right)^2}{2l}$ M1A1=A1ft

$\dfrac{1}{2}V^2 + 4gl = \dfrac{8}{9}gl + \dfrac{32}{9}gl$

$V^2 = \dfrac{8gl}{9}$ M1

solving for V^2

$V = \left(\dfrac{8gl}{9}\right)^{\frac{1}{2}}$ A1 7

or exact equivalents

(b) The maximum speed occurs when $a = 0$ B1

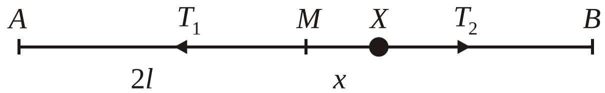
At M the particle is in equilibrium (the sum of the forces is zero)

$\Rightarrow a = 0$ B1 2

[9]

Alternative using Newton's second law.

(a)



HL $T_1 = \dfrac{4mi(l + x)}{l}$, $T_2 = \dfrac{4mg(l - x)}{l}x$

N2L $m\ddot{x} = T_2 - T_1 = -\dfrac{8mg}{l}x$ M1 A1

This is SHM, centre M

$a = \dfrac{l}{3}$, $\omega^2 = \dfrac{8g}{l}$ A1, A1ft

$v^2 = \omega^2(a^2 - x^2) \Rightarrow v^2 = \dfrac{8g}{l}\left(\dfrac{l^2}{9} - x^2\right)$ M1

Depends on showing SHM

At M, $x = 0$, $V^2 = \dfrac{8gl}{9}$, $V = \left(\dfrac{8gl}{9}\right)^{\frac{1}{2}}$ M1, A1 7

or exact equivalents

	(b)	The particle is performing SHM about the mid-point of AB.	B1	
		The maximum speed occurs at the centre of the oscillation (when $x = 0$)	B1	2

[9]

18.	(a)	KE loss + PE loss = EPE Gain		
		$\frac{1}{2}.m2gL + mg3L = \frac{\lambda(3L)^2}{2L}$	M1 A2(–1 e.e.)	
		$(*)\,\frac{8mg}{9} = \lambda$	A1	4
	(b)	$mg - T = m\ddot{x}$	M1 A1	
		$mg - \frac{8mg}{9L}(x+e) = m\ddot{x}$	M1 A1	
		$-\frac{8g}{9L}x = \ddot{x}$		
		Hence SHM about D	A1 c.s.o.	5
	(c)	(i) Period = $\frac{2\pi}{w} = 2\pi\sqrt{\frac{9L}{8g}} = 3\pi\sqrt{\frac{2}{2g}}$	M1 A1ft	
		(ii) mg = $\frac{8mg}{9L}e \Rightarrow e = \frac{9L}{8}$	B1	
		$9 = 3L - \frac{9L}{8} = \frac{15L}{8}$		
		$v_{max} = 9w = \frac{15L}{8}\sqrt{\frac{8g}{9L}}$	M1	
		$= \frac{5}{4}\sqrt{2gL}$	A1	5

[14]

19. Extn at bottom = $\dfrac{a}{\cos\alpha} - a = \dfrac{2a}{3}$ (0.67a or better) M1 A1

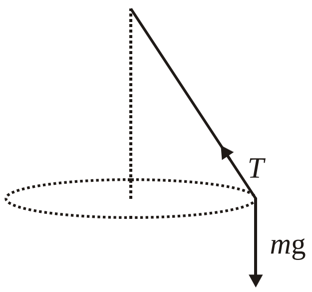
Energy: $mga \tan\alpha = \dfrac{2\lambda\left(\frac{2a}{3}\right)^2}{2a}$ M1 A1 A1 ft

$3mg = \lambda$ M1 A1

Second M0 if treated as equilibrium
Third M1 for solving for λ

[7]

20.



(a) (↕) $T\cos 60° = mg \Rightarrow T = 2mg$ * B1 1

(b) (↔) $T\sin 60° = mr\omega^2$ M1A1

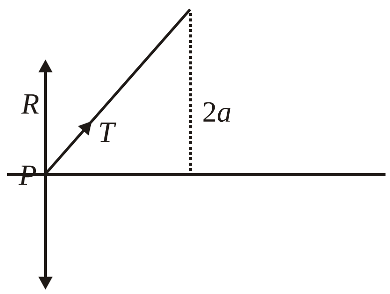
$r = L\sin 60°$ B1

$\omega = \sqrt{\dfrac{\mathbf{2g}}{\boldsymbol{L}}}$ A1 4

(c) Applying Hooke's Law: $2mg = \dfrac{\lambda}{(\frac{3}{5}L)}(L - \dfrac{3}{5}L)$; λ 3mg M1;A1 2

[7]

21. (a)



Length of string $(L) = \frac{10}{3}a$ B1

$\text{EPE} = \frac{\frac{1}{2}mg}{2a}(L-a)^2$ M1

$= \boldsymbol{\frac{49}{36}mga}$ A1 3

(b) Energy equation: $\frac{1}{2}mv^2 + \frac{\frac{1}{2}mg}{2a}a^2 = (\frac{49}{36}mga)_C$ M1A1ft

$v = \frac{2}{3}\sqrt{\boldsymbol{5ga}}$ or equivalent [allow $1.46\sqrt{ga}$, $\frac{14}{3}\sqrt{a}$] A1 3

(c) $T_V = T\sin\theta$ [implied by $R + T\sin\theta = mg$] B1

T_V in terms of θ [$\frac{mg}{2a}(\frac{2a}{\sin\theta} - a)\sin\theta$] M1

or in terms of x or AP [or $(\frac{mg(AP-a)}{2}.\frac{2a}{AP}$ or $\frac{mg}{2}\frac{x}{a}\frac{2a}{(a+x)}$]

(i) $T = \frac{1}{2}mg(2-\sin\theta)$ or $R = \frac{1}{2}mg\sin\theta$ A1
Complete method to show $R > 0$ M1A1
OR

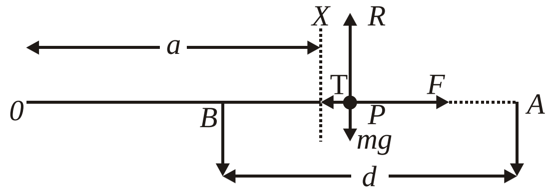
(ii) $T = \frac{1}{2}mg(2-\sin\theta)$; $mg(1 - \frac{a}{AP})$; $\frac{mgx}{a+x}$ A1
Complete method to show $T_V < mg$ or that $T_V \geq mg$ not poss M1A1
OR

(iii) $T = \frac{1}{2}mg(2-\sin\theta)$ A1
as θ increases T_V decreases; $T_V < T_{V\max} = \frac{7}{10}mg < mg$ M1A1 5

[In all cases: For A1 all working correct and arg. convincing]

[11]

22.



$F = \mu R = \mu mg$ — B1 B1

$\frac{4mg\,x^2}{2a}$

Attempt to relate Fd to EPE — M1

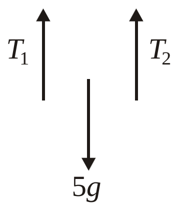
$\frac{2}{3}\ mg\,d = \dfrac{4mg(\frac{a}{2})^2}{2a}$ — M1 A1 ft

[A1 ft only on omission of g in F]

Final answer: $d = \frac{3}{4}a$ — A1 — 6

[6]

23.



$T_1 = \dfrac{175 \times 0.2}{1}$ — B1

$\dfrac{175 \times 0.2}{1} + \dfrac{\lambda \times 0.3}{0.9} = 49$ — M1 A 1

$\Rightarrow \lambda = 42$ — M1 A1 — 5

[5]

24. (a) $\dfrac{1}{2} \times \dfrac{58.8}{4} x^2 = 0.5 \times 9.8\,(x + 4)$ — M1 A1 A1

$3x^2 - 2x - 8 = 0$ — M1 A1

$(3x + 4)(x - 2) = 0,\ x = 2$

Distance fallen = 6 m — M1 A1 — 7

(b) $\dfrac{1}{2} \times 0.5v^2 = \dfrac{1}{2} \times \dfrac{58.8}{4} \times 3^2 - 0.5 \times 9.8 \times 3$ — M1 A1 A1

$v = 14.3\ \text{m s}^{-1}$ — M1 A1 — 5

[12]

25. $\dfrac{\lambda(100-l)}{l} = 0.3g$ M1 A1

$\dfrac{\lambda(110-l)}{l} = 0.5g$ A1

$\Rightarrow 5(100 - l) = 3(110 - l)$ M1

$l = 85$ cm A1 5

$\lambda = \dfrac{0.3g \times 85}{15} = 16.66$ N M1 A1 2

[7]

26. (a) Energy: $\frac{1}{2} \times 2 \times 10^2 = 2 \times 9.8 \times h + \frac{1}{2} \times \dfrac{120 \times h^2}{3}$ M1 A1 A1

$20h^2 + 19.6h - 100 = 0$ M1

$h = \dfrac{-19.6 \pm \sqrt{(19.6^2 + 4 \times 20 \times 100}}{40}$ M1

$= 1.7991\ldots \approx 1.8$ (or 1.80) m A1 6

(b) $\frac{1}{2} \times 2 \times V^2 = 2 \times 9.8 \times 1.8 + \frac{1}{2} \times \dfrac{120 \times 2.3^2}{3} - \frac{1}{2} \times \dfrac{120 \times 0.5^2}{3}$ M1 A1 A1

$V = 11.7$ (3 s.f.) or 12 (2 s.f.) m s^{-1} M1 A1 5

[11]

1. This was probably the least well done of all the questions and correct solutions were relatively rare. The most common mistake was the assumption that there was no final EPE but this was often combined with other errors to give a huge variety of different wrong answers. A surprisingly large proportion of candidates treated this as an equilibrium question, either starting with $T = \mu R + mg \sin \theta$ or slipping an EPE term in as well for good measure. Others realised that it was an energy question but forgot to include the work done against friction; these attempts either used only the frictional force in their equation or ignored it completely, offering as their solution "Initial EPE = *mgh*". Another common error was to include the GPE term twice, once as energy and again as part of the "Work done" expression, showing a lack of understanding of the origin of the *mgh* formula. Very many candidates scored only the 3 marks for finding friction, while those who thought that this was a simple conversion of EPE into GPE had no need to find the friction and so didn't even earn these. Some candidates who included all necessary terms fell at the accuracy hurdle. Inexplicably, a final extension/ compression of 0.2 was not uncommon and other errors arose from inappropriate use of the various lengths mentioned, 1.5, 0.9 and 0.7. There were also all the usual sign errors generated by mistakes in identifying gains and losses. A few candidates produced a perfect solution but lost the final mark by giving their answer as 12.7008.

2. The majority gained full marks for part (a) but some used m instead of $2m$ for the mass and others forgot to complete the question by adding $3a$ to their extension. However, part (b) was a different story. This was a standard question and should have been routine for most but was very poorly done. Some candidates measured the extension from the natural length, which can produce a correct result provided the appropriate substitution is employed. Probably they were not fully aware of what they were doing and so stopped work when they did not reach the required equation. Many had inconsistent masses, with $2mg$ and $m\ddot{x}$ appearing in the same line of working, although some realised and backtracked successfully. The omission of $2mg$ in the equation of motion led to the correct answer when the extension was incorrectly measured from the natural length so candidates assumed that they had answered correctly. Poor notation was often seen; "acc." or "*a*" was used for acceleration (as well as its length application) and e was used for the variable extension along with an acceleration $\ddot{x}$. There were many sign errors seen in the equations and although some candidates realised they had made errors and either corrected their work or fiddled the result, others seemed to think that the equation $\ddot{x} = \omega^2 x$ proved S.H.M. Good attempts were presented for parts (c) and (d) although some thought that the amplitude was 3a/4; $\omega = \sqrt{\frac{a}{g}}$ was another common error. Some forgot to complete their work in part (d) by adding $\frac{a}{8}$ and $\frac{3a}{128}$ to obtain the final answer.

3. This was probably the best answered question on the paper. Most candidates obtained T (not always shown explicitly) by resolving in two directions. There were a few who drew a triangle of forces. Good candidates were able to “see” the triangle without working and wrote down immediately that $T = 50$. A few unfortunately thought they were dealing with 40g and 30 even though was it clearly stated in the question that the weight was 40 N and not that the mass was 40 kg. The most common mistake was in the arithmetic – $50 = \frac{\lambda x}{0.5}$ so $\lambda x = 100$! Very few forgot to complete their solution by adding 0.5 to their extension to find OP.

4. Most candidates managed to arrive at the required result in part (a), though some unnecessarily split the motion into two parts, considering freefall initially to find the kinetic energy when the string became taut and then proceeding to consider the taut string and others would clearly have failed had not the answer been provided.
Parts (b) and (c) were often difficult to disentangle. Some candidates took an SHM approach from the start of to the Examiner”. The main fault was not when to start considering SHM but not establishing a correct equation to prove that the motion was SHM; no credit is given for making assumptions of this nature. A fully correct solution using SHM was rare, the equations frequently being unsatisfactory due to using x for the distance from the equilibrium point and confusing it with x as defined in the question to be the extension of the string.
For the non-SHM solutions, in part (b) many candidates assumed that the maximum speed occurred when $x = 0$ rather than when $a = 0$. In part (c) most substituted $v = 0$ in the result from part (a). Some did not expect to obtain a quadratic and so stopped working (or ran out of time?). Of those who obtained a solution for their quadratic equation, some would then try incorrectly to use their value for x as the amplitude in SHM instead of using an equation of motion and Hooke’s law. Many equations of motion omitted the weight of the particle.

5. This was a straightforward opening question but many candidates spoiled their solutions with careless errors. Most managed to use Pythagoras to calculate the length of the extended string. However some appeared to have misinterpreted the information given in the question and took the natural length as 12 m instead of 8 m. Hooke’s law was well known but problems arose as candidates were confused between the full string and half strings, with some thinking the tensions in these were different. Significant numbers gave the mass as their final answer instead of the weight as demanded. Part (b) gave rise to fewer problems. The formula for the elastic potential energy was known by all but a small number of candidates and many who had an incorrect extension in part (a) either completely recovered to gain full marks or gained 2 of the 3 marks by follow through.

6. Part (a) was usually handled confidently with Hooke's Law in the equilibrium position applied to each spring and the resulting tensions equated. There were some unfortunate careless errors such as "total extension = 5 – 2 – 1 = 3" which appeared far too often. Mistakes in the extensions led to difficulties in the later parts of the question but some able candidates didn't think to check their extensions when their confidently applied method in (b) failed to work as expected. The work seen in part (b) suggested that many candidates are not aware that to prove SHM they need an equation of motion that reduces to the form $\ddot{x} = -\omega^2 x$; some even tried to give a written explanation of the motion, with no equations provided at all. Even if the question had not told them that the equilibrium position was the centre of the oscillation, candidates should have known this and measured their displacement from this point; many chose some other point instead. There were also numerous sign errors in the equation of motion, often "corrected" but not always in a valid manner.

Lack of success in part (b) meant many candidates had no suitable value for ω to use in part (c). Some were content to invent a number and continue, thereby making the method and follow through marks available, others gave up, though this may have been due to lack of time. Most who worked on this part realised that the amplitude could be obtained using max $v_{\max} = a\omega$ and hence the time required using $x = a \sin\omega t$. Solutions using $x = a\cos\omega t$ were seen occasionally but the extra work needed to obtain the necessary time was often omitted. The instruction to find the *proportion* of the time was usually ignored completely resulting in the loss of three marks as the period was not calculated since the necessity for it was not apparent.

7. Most candidates resolved horizontally and vertically in (a) and then found the tangent of their angle between the string and the horizontal or vertical before proceeding to obtain the tension. It was not uncommon to see solutions which started with the Pythagoras equation in line 3 of the mark scheme. This is a risky approach as errors in this equation cause many marks to be lost if the equation is not derived from the resolving equations. Hooke's law and the formula for elastic potential energy were well known and frequently applied correctly in (b) to reach a correct answer. However, omission of some or all of the letters *m*, *g* and a in the final answer was fairly common.

8. Many candidates used unduly complicated methods for both parts of this question. Some tried to use S.H.M. but very few attempted to establish the motion as being S.H.M. before quoting and using the standard formulae. In part (a) there was confusion between equilibrium and rest positions. Some candidates used Hooke's law and resolved parallel to the plane, finding the equilibrium extension and claimed that the particle was at rest at this point. They then proceeded to use the same extension (now correctly) in (b) to obtain the greatest speed. Energy methods often had a missing term or a gravitational potential energy term which was inconsistent with the positions involved. Alternative methods using Newton's second law and calculus were reasonably popular but many were incomplete.

9. Part a was generally well done although a few left *m* in their final answer.
Most answered part (b) correctly but several used $T \sin \theta$ instead of $T \cos \theta$ as the resolved part of T in the vertical direction, and others used $\cos \theta = 0.56/0.72$ which was not appropriate.

10. There were some excellent solutions but also a great many inaccurate and muddled attempts. Although the theory was well known, it was very common for one or more energy terms to be left out (most commonly the initial EPE or the GPE). Confusion between energy losses and gains often led to wrong signs, a difficulty which is easily dealt with by equating initial energy to final energy instead. Many candidates seemed unclear about the relative positions they were considering and there was a great deal of inaccuracy in identifying extensions, heights and positions of zero KE. In part (a) it was usually possible to follow the candidate's line of reasoning and identify the mistakes but many of the attempts at (b) used inconsistent initial energy values from a variety of locations and failed to follow the 'Advice to candidates' that their methods should be made clear to the examiner. The clearest solutions started with a very well labelled diagram showing all the significant points and identifying the extension associated with each. Statements such as "Energy at A = …..., energy at B = ……" helped to avoid muddled values and usually led to correct or nearly correct equations. By contrast, a great many attempts at (b) contained no statement at all but simply an equation of unidentifiable terms.

11. Part (a) was well done. A correct use of Hooke's Law, combined with vertical resolution and the geometry of the situation, was the standard approach. A few made the error of not realising that there were two tensions for the resolution but only one for Hooke's Law. Many good arguments were seen in part (b). There were two main methods; using conservation of energy with three terms and showing $v = 0$, and finding the elastic potential energy loss and showing that it was equal to the gain in gravitational potential energy at the level of AB. There was a tendency for candidates to drop the l in their lengths. As energy is involved, this gave dimensionally incorrect solutions but candidates who did this could usually gain 4 of the 6 marks if their solution was otherwise correct.

12. Some excellent solutions were seen here with many gaining full marks. Of those who did not, some made a small slip in the processing (or occasionally in the signs of the work–energy equation, though such mistakes were rarer than might have been expected); weaker candidates made more fundamental errors, e.g. equating energy with force.

13. Part (a) was almost universally correct. Part (b) caused considerable problems: many assumed that they could find the acceleration as a (unspecified) 'a', and that if they showed that this was equal to '$-196x$' they had succeeded in showing that the motion was SHM. Such candidates failed to realise that any acceleration given as an unspecified a needs its direction clearly specified. Hence, without the use of an expression for $\ddot{x}$ as equal to $-196x$, they could make no progress. Weaker candidates also failed to see that the equation of motion had to include the weight as well. In parts (c) and (d) a common mistake was effectively to assume that the particle came to rest at the end of an oscillation within the simple harmonic motion. Nevertheless, more able candidates were able to complete the question accurately and overall, this proved to be a good discriminating question for the final one on the paper.

14. Part (a) was well done. The only common error was considering the elastic potential energy in only one part of the string instead of in both parts. Most candidates realised that energy was involved and the few who attempted using Newton's Second Law almost all failed to consider a general point of the motion and so gained no credit. Nearly all candidates could start part (b) by resolving vertically and writing down some form of Hooke's Law. The manipulations required to obtain the required trigonometric relation, however, were demanding and even strong candidates often needed two or three attempts to complete this and the time spent on this was sometimes reflected in an inability to complete the paper. This was particular the case if candidates attempted to use or gain information by writing down an equation of energy. This leads to very complicated algebra and is not a practical method of solving questions of this type at this level. (Correctly applied it leads to a quartic not solvable by elementary methods.) For those who were successful in part (a), writing T in terms of , say, the angle made by each part of the string with the vertical proved the critical step. If they obtained $T = \frac{49}{0.75}\left(\frac{0.75}{\sin a} - 0.75\right)$, or its equivalent, the majority of candidates had the necessary trigonometric skills to complete the question.

15. This provided a fairly straightforward start and most scored well but there were fewer full marks than expected. The answer to (a) was often given as 13.58, losing a mark, the value of F was frequently used mistakenly for T in (b) and accuracy errors were fairly common in (c) when a 2 sig fig answer to (b) was used to calculate a 3 sig fig value for the energy.

16. Apart from (a), which was almost always correct, this was not well done. With the definition ofclearly defined in the question, the proof in (b) should have been straightforward. For many it was, but there were again a huge number of fiddled attempts. Often, candidates started out correctly with $mg - T = \pm ma$, only to cross out the mg subsequently, when they realised that $T = \lambda x / l = ma$ led to the required expression, and therefore lose the mark for this equation.

Part (c) was not well done either. Most chose to use the formula $v^2 = \omega^2(a^2 - x^2)$ but values for a and x were not often correct. Use of energy was popular as well but many omitted either the EPE or the GPE. Part (d) was a great discriminator and only the best candidates were able to describe the transition from SHM to free motion under gravity to SHM clearly and accurately.

17. This proved the hardest question on the paper and many scored no marks. The use of energy was seen more frequently than approaches using Hooke's Law and an equation of motion, but energy was not well handled. There are four elastic energies involved in the question; two different energies in the initial position and two identical energies at the mid-point. It was common to see three of these four energies omitted. When it was realised that there were two energies at the mid-point, it was not unusual for candidates to think these "cancelled" out. Those using Hooke's Law again often only had one string in tension and no credit can be gained using SHM methods unless a variable extension is used. With the SHM method, candidates often had difficulty finding the extension of the two strings but, on balance, this method tended to be used more successfully than energy. It is worth recording that a few candidates produced completely correct solutions, using Newton's Second Law, measuring the displacements from *A*, *B* or the initial position of *P*. They then used methods of solving second order differential equations to complete the solution without reference to SHM or any further mechanical principles. This module is probably now taken by a high proportion of candidates who have studied Further Pure Mathematics and such solutions may be more commonly seen than in the past. Questions requiring explanation remain unpopular with candidates. In this case, the comments expected were that the maximum speed occurs when the acceleration is zero and the mid-point is the position of equilibrium and, hence, the acceleration there is zero.

18. Most realised that the first part required the use of energy and were able to obtain the required result. There was a disappointing response to the "standard proof" in part (b); many candidates simply ignored the weight and scored no marks. Part (c) (i) was well done but there were few correct solutions to (ii), where most were unable to find the amplitude of the oscillation.

19. Even among able candidates, this was rarely recognised as a question about energy, with most attempts assuming that the lowest point reached would be the equilibrium position. Consequent use of $2T\sin\alpha = mg$ and Hooke's law lost 5 of the 7 available marks leaving most candidates able to score only the 2 marks for finding the extension. This was often correct but, as in question 1, there were many numerical and algebraic slips. Some did not even realise that the extension could be found, and persisted with an extension *x*. Those who did correctly use an energy method to solve the problem often produced concise and accurate solutions but a significant number forgot to include the EPE from both strings.

20. Although the answer was given in part (a) it is still good to report that all but a handful of candidates gained the mark. Part (b) was generally well answered, although $T = mr\omega^2$, $T\sin 60° = mL\omega^2$, and errors in eliminating *r*, were occasionally seen. Probably the most common error in this question occurred in part (c), where *L*, instead of $\frac{3}{5}L$, was often seen in the denominator of Hooke's law.

21. Although in part (a) some candidates did not find the correct value for the extension of the string, and in part (b) the elastic energy in the string when it was vertical was sometimes omitted, these two parts were generally answered well.

There were some very impressive solutions, showing good manipulative and deductive skills, by the most able candidates in part (c), but this proved to be the most challenging part of the paper and the modal score was one mark. Many candidates did not follow the instruction, and the hint, to consider the vertical component of the string at a general point.

The most common mistake was to assume that the maximum value of T being at the point of release, and that it remains on the table at that point, was sufficient justification for the particle to remain on the table during the motion. The fact that, during the motion, as T decreased so $\sin\theta$ increased and so the behaviour of $T\sin\theta$ was not obvious, was not appreciated by the majority of candidates.

22. Good candidates found this straightforward, finding AB immediately from a single application of the work-energy equation. However, a large number of candidates did not have a clear strategy for the problem and marks of 1, 2 or 3 were common.

Common errors were: using $\frac{3a}{2}$, not the *extension*, in the energy term; omitting the frictional force, or equating it to the tension, or including the tension when considering the work done, e.g. $(T - F)x = \dfrac{4mg\left(\frac{a}{2}\right)^2}{2a}$; and not realising that the string was slack at B, so introducing an extra " elastic energy" term.

Candidates who used a two-stage approach, first finding the velocity at the instance the string became slack, were rarely successful.

23. This proved to be a straightforward starter for most and full marks was often seen. The most common error was to assume that the tensions in the two strings are equal but there were other odd errors in use of Hooke's Law and/or arithmetic.

24. Candidates who considered the energy changes from the start to the finish were generally much more successful than those who considered intermediate points. A significant number tried to use SHM formulae but were given no credit for it unless they had first proved that the motion was simple harmonic. Some also tried to use a differential equation but usually without success. Many thought, in part (b), that the string went slack at the equilibrium position and a few lost the last mark for giving their final answer to more than 3 SF. This was deemed inappropriate as the answer involved taking g as 9.8.

25. No Report available for this question.

26. No Report available for this question.